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(*-----Filename: powder_MAS.nb-----*)
(* The main function powderMAS[] provides the line intensity
of the central transition (if quanta = 1) or that of the -3-quantum
transition (if quanta = 3) of a spin I = 3/2 system rotating at the
magic angle, submitted to the first-order quadrupole interaction
(if order = 1) or the first- and second-order quadrupole interactions
(if order = 2), and excited by an x-pulse.
This line intensity depends on
(1) the rotor spinning speed VrotkHz (in kHz unit),
(2) the quadrupole coupling constant QCCMHz (in MHz unit),
(3) the asymmetry parameter η,
(4) the three Euler angles αd, βd, and γd (in degree unit) orienting
the rotor in the principal-axis system of the EFG tensor ΣPAS,
(5) the Larmor frequency ω0MHz (in MHz unit),
(6) the strength of the radiofrequency field ωRFkHz (in kHz unit),
(7) the pulse duration increasing from 0 to tf (in μs unit)
by step of tau (in μs unit),
(8) the number maxα of summation steps of the Euler angle α
in the 0 2π rang,
(9) the number maxβ of summation steps of the Euler angle β
in the 0 π rang,
(10) the number maxγ of summation steps of the Euler angle γ
in the 0 2π rang.

The main function provides the parameters ai, bi, a2i, b2i,
a4i, and b4i for an orientation of the rotor to the sub-function f[].

The sub-function f[] provides the density matrix ρ(t) via
the value of ωQ (if order = 1), and of ωQ21 and ωQ22 (if order = 2)
by taking into account the rotor spinning speed. The spin system is
supposed to be time-independent during each duration Δt or tau.
It returns Table s[m] to the main function powderMAS[]. *)
(*-----*)
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(*-----*)
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(* Sub-function f[] *)
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$$f[\text{order}_, \text{QCC}_, \omega\text{RF}_, \Delta t_, n_] := \left( \right.$$

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For [m = 1, m ≤ n, m++,
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$$\omega_Q = \frac{\text{QCC}}{2\sqrt{6}} \left( \right.$$

$$+ d_{1c} \cos[(m-1)\Delta t * \omega_{rot}] + d_{2c} \cos[(m-1) * 2\Delta t * \omega_{rot}] \\ + d_{1s} \sin[(m-1)\Delta t * \omega_{rot}] + d_{2s} \sin[(m-1) * 2\Delta t * \omega_{rot}];$$

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ωQ21 = 0;   ωQ22 = 0 ;
If [order == 2, {
  W20 = d21c Cos[(m - 1) Δt * ωrot] + d22c Cos[(m - 1) * 2 Δt * ωrot]
      + d21s Sin[(m - 1) Δt * ωrot] + d22s Sin[(m - 1) * 2 Δt * ωrot];
  W40 = a40
      + d44s Sin[(m - 1) * 4 Δt * ωrot] + d44c Cos[(m - 1) * 4 Δt * ωrot]
      + d43s Sin[(m - 1) * 3 Δt * ωrot] + d43c Cos[(m - 1) * 3 Δt * ωrot]
      + d42s Sin[(m - 1) * 2 Δt * ωrot] + d42c Cos[(m - 1) * 2 Δt * ωrot]
      + d41s Sin[(m - 1) * Δt * ωrot] + d41c Cos[(m - 1) * Δt * ωrot];
  ωQ21 =  $\frac{-1}{\omega_0} \frac{QCC^2}{36} \left( \frac{-21}{2\sqrt{70}} W_{40} + \frac{9}{2\sqrt{5}} W_{00} \right)$ ;
  ωQ22 =  $\frac{-1}{\omega_0} \frac{QCC^2}{36} \left( \frac{27}{2\sqrt{70}} W_{40} + \frac{6}{\sqrt{14}} W_{20} - \frac{3}{2\sqrt{5}} W_{00} \right)$ ;
}]; (* End of If order == 2 *)

Ha =  $\begin{pmatrix} \omega_Q + \omega_{Q21} & -\frac{\sqrt{3}}{2} \omega_{RF} & 0 & 0 \\ -\frac{\sqrt{3}}{2} \omega_{RF} & -\omega_Q + \omega_{Q22} & -\omega_{RF} & 0 \\ 0 & -\omega_{RF} & -\omega_Q - \omega_{Q22} & -\frac{\sqrt{3}}{2} \omega_{RF} \\ 0 & 0 & -\frac{\sqrt{3}}{2} \omega_{RF} & \omega_Q - \omega_{Q21} \end{pmatrix}$ ;

{HT, Tp} = Eigensystem[N[Ha]];          T = Transpose[Tp];
n1 = DiagonalMatrix[Exp[-i * Δt * HT]];
ρ1 = T.n1.Tp;      ρ2 = T.Conjugate[n1].Tp;      ρ0 = ρ1.ρ0.ρ2;
s[m] = ρ0;
]; (* End of For m *)

); (* End of sub-function f[] *)

(* Main function powderMAS[] *)
powderMAS[order_, ω0Mhz_, QCCMhz_, ωRFkHz_,
  VrotkHz_, tf_, tau_, η_, maxα_, maxβ_, maxγ_, quanta_] := {
  ω0 = ω0Mhz * 2 π * 103;      QCCbis = QCCMhz * 2 π * 103;      ωbRFbis = ωRFkHz * 2 π;
  ωrot = VrotkHz * 2 π;      Δt = tau * 10-3;      ns = tf / tau;
  W00 = ( $\sqrt{5} / 10$ ) (3 + η2);

  (* Table h stores the line intensity for each pulse duration *)
  For [i = 0, i ≤ ns, i++, h[i] = 0]; (* Clear Table h *)

  For [j = 0, j < maxα, j++, Print[j]; (* Summation on Euler angle α ∈ [0, 2 π[ *)
  For [k = 1, k < maxβ, k++, (* Summation on Euler angle β ∈ ] 0, π[ *)
    (* probability sin 0° = sin 180° = 0 *)
  For [c = 0, c < maxγ, c++, (* Summation on Euler angle γ ∈ [0, 2 π[ *)
    {(* Thermodynamic equilibrium of the density matrix *)

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$$\rho_0 = \text{DiagonalMatrix}\left[\left\{\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right\}\right];$$

$$c_\alpha = \text{Cos}\left[\frac{2\pi j}{\text{max}\alpha}\right]; \quad s_\alpha = \text{Sin}\left[\frac{2\pi j}{\text{max}\alpha}\right]; \quad c_{2\alpha} = 2c_\alpha^2 - 1; \quad s_{2\alpha} = 2c_\alpha s_\alpha;$$

$$c_\beta = \text{Cos}\left[\frac{\pi k}{\text{max}\beta}\right]; \quad s_\beta = \text{Sin}\left[\frac{\pi k}{\text{max}\beta}\right]; \quad c_{2\beta} = 2c_\beta^2 - 1; \quad s_{2\beta} = 2c_\beta s_\beta;$$

$$c_\gamma = \text{Cos}\left[\frac{2\pi c}{\text{max}\gamma}\right]; \quad s_\gamma = \text{Sin}\left[\frac{2\pi c}{\text{max}\gamma}\right]; \quad c_{2\gamma} = 2c_\gamma^2 - 1; \quad s_{2\gamma} = 2c_\gamma s_\gamma;$$

(\* Coefficients  $a_i$  and  $b_i$  involved in  $V_{(2,0)}$  \*)

$$\begin{aligned} a_1 &= -\eta s_{2\alpha} s_\beta / \sqrt{3}; & b_1 &= -(-3 + \eta c_{2\alpha}) s_{2\beta} / (2\sqrt{3}); \\ a_2 &= -\eta c_\beta s_{2\alpha} / \sqrt{6}; & b_2 &= -(\eta c_{2\alpha} (3 + c_{2\beta}) + 6 s_\beta^2) / (4\sqrt{6}); \\ d_{2s} &= a_2 c_{2\gamma} + b_2 s_{2\gamma}; & d_{1s} &= a_1 c_\gamma + b_1 s_\gamma; \\ d_{2c} &= a_2 s_{2\gamma} - b_2 c_{2\gamma}; & d_{1c} &= a_1 s_\gamma - b_1 c_\gamma; \end{aligned}$$

If [order == 2, {

$$\begin{aligned} c_{4\alpha} &= 2c_{2\alpha}^2 - 1; \\ c_{4\beta} &= 1 - 8c_\beta^2 (1 - c_\beta^2); & s_{4\beta} &= 4c_\beta c_{2\beta} s_\beta; \\ c_{3\gamma} &= c_\gamma (4c_\gamma^2 - 3); & s_{3\gamma} &= s_\gamma (4c_\gamma^2 - 1); \\ c_{4\gamma} &= 2c_{2\gamma}^2 - 1; & s_{4\gamma} &= 4c_\gamma c_{2\gamma} s_\gamma; \end{aligned}$$

(\* Coefficients  $a_{2i}$  and  $b_{2i}$  involved in  $W_{(2,0)}$  \*)

$$\begin{aligned} a_{22} &= -\sqrt{2/7} \eta c_\beta s_{2\alpha}; & b_{22} &= -(\eta c_{2\alpha} (3 + c_{2\beta}) + s_\beta^2 (-3 + \eta^2)) / (2\sqrt{14}); \\ a_{21} &= -(2/\sqrt{7}) \eta s_\beta s_{2\alpha}; & b_{21} &= (-3 - 2c_{2\alpha} \eta + \eta^2) s_{2\beta} / (2\sqrt{7}); \\ d_{22s} &= a_{22} c_{2\gamma} + b_{22} s_{2\gamma}; & d_{21s} &= a_{21} c_\gamma + b_{21} s_\gamma; \\ d_{22c} &= a_{22} s_{2\gamma} - b_{22} c_{2\gamma}; & d_{21c} &= a_{21} s_\gamma - b_{21} c_\gamma; \end{aligned}$$

(\* Coefficients  $a_{4i}$  and  $b_{4i}$  involved in  $W_{(4,0)}$  \*)

$$\begin{aligned} a_{40} &= \frac{-\sqrt{7/10}}{2304} \\ & \quad ((18 + \eta^2) (9 + 20c_{2\beta} + 35c_{4\beta}) + 240\eta c_{2\alpha} (5 + 7c_{2\beta}) s_\beta^2 + 280\eta^2 c_{4\alpha} s_\beta^4); \\ a_{41} &= (\sqrt{5/7} / 72) \eta s_{2\alpha} s_\beta (15 + 21c_{2\beta} + 14\eta c_{2\alpha} s_\beta^2); \\ b_{41} &= (\sqrt{5/7} / 288) ((-18 - \eta^2 - 12\eta c_{2\alpha} + 7\eta^2 c_{4\alpha}) s_{2\beta} - 7(-3 + \eta c_{2\alpha})^2 s_{4\beta}); \\ a_{42} &= -(\sqrt{5/14} / 18) \eta c_\beta s_{2\alpha} (-9 + 21c_{2\beta} + 14\eta c_{2\alpha} s_\beta^2); \\ b_{42} &= \frac{-1}{72} \sqrt{5/14} \\ & \quad (3\eta c_{2\alpha} (5 + 4c_{2\beta} + 7c_{4\beta}) + (7\eta^2 c_{4\alpha} (3 + c_{2\beta}) + (18 + \eta^2) (5 + 7c_{2\beta})) s_\beta^2); \\ a_{43} &= -(\sqrt{35} / 72) \eta (-3 - 9c_{2\beta} + \eta c_{2\alpha} (5 + 3c_{2\beta})) s_{2\alpha} s_\beta; \\ b_{43} &= -(\sqrt{35} / 288) (-18 - \eta^2 - 12\eta c_{2\alpha} + 7\eta^2 c_{4\alpha} + 2(-3 + \eta c_{2\alpha})^2 c_{2\beta}) s_{2\beta}; \\ a_{44} &= -(\sqrt{35/2} / 72) \eta c_\beta s_{2\alpha} (\eta c_{2\alpha} (3 + c_{2\beta}) + 6s_\beta^2); \\ b_{44} &= -\frac{\sqrt{35/2}}{2304} (\eta^2 c_{4\alpha} (35 + 28c_{2\beta} + c_{4\beta}) + 48\eta c_{2\alpha} (3 + c_{2\beta}) s_\beta^2 + 8(18 + \eta^2) s_\beta^4); \\ d_{41s} &= a_{41} c_\gamma + b_{41} s_\gamma; & d_{41c} &= a_{41} s_\gamma - b_{41} c_\gamma; & d_{42s} &= a_{42} c_{2\gamma} + b_{42} s_{2\gamma}; \\ d_{42c} &= a_{42} s_{2\gamma} - b_{42} c_{2\gamma}; & d_{43s} &= a_{43} c_{3\gamma} + b_{43} s_{3\gamma}; & d_{43c} &= a_{43} s_{3\gamma} - b_{43} c_{3\gamma}; \\ d_{44s} &= a_{44} c_{4\gamma} + b_{44} s_{4\gamma}; & d_{44c} &= a_{44} s_{4\gamma} - b_{44} c_{4\gamma}; \end{aligned}$$

]; (\* End of If order == 2 \*)

f[order, QCCbis, wBRFBis, Δt, ns]; (\* Call the sub-function f[] \*)

For [i = 1, i ≤ ns, i++,

(\* Normalized central-transition line intensity \*)

If [quanta == 1, {s[i] = Im[s[i][[3, 2]]], h[i] = h[i] + s<sub>β</sub> s[i] \* 2 / 5};

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      (* -3-quantum line intensity *)
      If [quanta == 3, {s[i] = Im[s[i][[4, 1]]], h[i] = h[i] + S $\beta$  s[i]};
    ]; (* End of For i *)

  }]; (* End of For c *)
]; (* End of For k *)
]; (* End of For j *)

For [i = 1, i ≤ ns, i++, h[i] = h[i] / (max $\alpha$  (max $\beta$  - 1) max $\gamma$ );];
(* Powder normalization *)

(*----- Provide Table crystalMAS containing -----*)
(*-----pulse duration t and line intensity -----*)
Print["*****"];
For[a = 0, a ≤ ns, a++, time[a] = a * tau;];
powderMAS = Chop[Table[{tt, time[tt], NumberForm[h[tt], 10]}, {tt, 0, ns}]];
Print[TableForm[powderMAS,
  TableHeadings -> {None, {"Rang", "t ( $\mu$ s)", "intensity"}}]];
(*----- Graph display -----*)
Print["*****"];
ListPlot[Table[{tt * tau, h[tt]}, {tt, 0, ns}],
  PlotJoined -> True,
  PlotLabel -> "Int=f(t)",
  AxesLabel -> {"t ( $\mu$ s)", "Int. (U.A.)"},
  PlotStyle -> {Hue[0.1]},
  TextStyle -> {FontFamily -> "Times", FontSize -> 12}];
); (* End of main function powderMAS *)

(* Call the main function with the corresponding numerical parameters *)
powderMAS[ 2 , 105.8731007, 8 ,
  100 , 15 , 20, 1.0 , -1, 2 , 3 , 3 , 1 ];
(* powderMAS[order_,  $\omega$ 0Mhz_, QCCMHz_,  $\omega$ RFkHz_,
  VrotkHz_,tf_, tau_,  $\eta$ _,max $\alpha$ _,max $\beta$ _,max $\gamma$ _,quanta_] *)

(*-----*)
(* Table powderMAS.m in Microsoft EXCEL format *)
(*-----*)
Clear[writeExcel];
writeExcel[filename_String, data_List] :=
  Module[ { file = OpenWrite[filename] },
    Scan[(
      WriteString[file, First[#]];
      Scan[
        WriteString[file, "\t", #] &,
        Rest[#]
      ]; (* End of Scan *)
      WriteString[file, "\n"]
    ) &,
    data
  ]; (* End of Scan *)
  Close[file]
]; (* End of Module *)

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writeExcel["powderMAS.m", powderMAS];  
  
Remove[order,  $\omega$ 0Mhz, QCCMHz,  $\omega$ RFkHz, VrotkHz,  
  tf, tau,  $\eta$ , max $\alpha$ , max $\beta$ , max $\gamma$ , QCCbis,  $\omega$ BRFbis, ns,  
   $\alpha$ ,  $\beta$ ,  $\gamma$ , i, j, k, c, h,  $\rho$ 0, f, a, powderMAS,  
  n, s, m,  $\omega$ Q, Ha, T, Tp, HT, n1,  $\omega$ Q21,  $\omega$ Q22]
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