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(*-----Filename: crystal_MAS.nb-----*)
(* The main function crystalMAS[] provides the line intensity
of the central transition (if quanta = 1) or that of the -3-quantum
transition (if quanta = 3) of a spin I = 3/2 system rotating at the
magic angle, submitted to the first-order quadrupole interaction
(if order = 1) or the first- and second-order quadrupole interactions
(if order = 2), and excited by an x-pulse.
This line intensity depends on
(1) the rotor spinning speed VrotkHz (in kHz unit),
(2) the quadrupole coupling constant QCCMHz (in MHz unit),
(3) the asymmetry parameter η,
(4) the three Euler angles αd, βd, and γd (in degree unit) orienting
the rotor in the principal-axis system of the EFG tensor ΣPAS,
(5) the Larmor frequency ω0Mhz (in MHz unit),
(6) the strength of the radiofrequency field ωRFkHz (in kHz unit),
(7) the pulse duration increasing from 0 to tf (in μs unit)
by step of tau (in μs unit).

The main function provides the parameters ai, bi, a2i, b2i,
a4i, and b4i for an orientation of the rotor to the sub-function f[]).

The sub-function f[] provides the density matrix ρ(t) via
the value of ωQ (if order = 1), and of ωQ21 and ωQ22 (if order = 2)
by taking into account the rotor spinning speed. The spin system is
supposed to be time-independent during each duration Δt or tau.
It returns Table s[m] to the main function crystalMAS[]). *)
(*-----*)
(* Author: Redouane HAJJAR
Address: Université Pierre et Marie Curie-Paris 6, UMR7142 (CNRS),
Laboratoire des Systèmes Interfaciaux à l' Echelle Nanométrique,
4 place Jussieu,casier 196, Paris,F-75005, FRANCE *)
(*-----*)
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(* Sub-function f[] *)
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f[order_, QCC_, ωRF_, Δt_, n_] :=
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For [m = 1, m ≤ n, m++,
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$$\omega Q = \frac{QCC}{2\sqrt{6}} * ($$

$$d_{1c} \text{Cos}[(m-1) \Delta t * \omega rot] + d_{2c} \text{Cos}[(m-1) * 2 \Delta t * \omega rot]$$

$$+ d_{1s} \text{Sin}[(m-1) \Delta t * \omega rot] + d_{2s} \text{Sin}[(m-1) * 2 \Delta t * \omega rot]);$$

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ωQ21 = 0; ωQ22 = 0;
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If [order == 2, {
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$$W_{20} = d_{21c} \text{Cos}[(m-1) \Delta t * \omega rot] + d_{22c} \text{Cos}[(m-1) * 2 \Delta t * \omega rot]$$

$$+ d_{21s} \text{Sin}[(m-1) \Delta t * \omega rot] + d_{22s} \text{Sin}[(m-1) * 2 \Delta t * \omega rot];$$

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W40 = a40
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$$+ d_{44s} \text{Sin}[(m-1) * 4 \Delta t * \omega rot] + d_{44c} \text{Cos}[(m-1) * 4 \Delta t * \omega rot]$$

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+ d43s Sin[(m - 1) * 3 Δt * ωrot] + d43c Cos[(m - 1) * 3 Δt * ωrot]
+ d42s Sin[(m - 1) * 2 Δt * ωrot] + d42c Cos[(m - 1) * 2 Δt * ωrot]
+ d41s Sin[(m - 1) Δt * ωrot] + d41c Cos[(m - 1) Δt * ωrot];

ωQ21 =  $\frac{-1}{\omega_0} \frac{QCC^2}{36} \left( \frac{-21}{2\sqrt{70}} W_{40} + \frac{9}{2\sqrt{5}} W_{00} \right)$ ;
ωQ22 =  $\frac{-1}{\omega_0} \frac{QCC^2}{36} \left( \frac{27}{2\sqrt{70}} W_{40} + \frac{6}{\sqrt{14}} W_{20} - \frac{3}{2\sqrt{5}} W_{00} \right)$ ;
}]; (* End of If order == 2 *)

Ha =  $\begin{pmatrix} \omega_Q + \omega_{Q21} & -\frac{\sqrt{3}}{2} \omega_{RF} & 0 & 0 \\ -\frac{\sqrt{3}}{2} \omega_{RF} & -\omega_Q + \omega_{Q22} & -\omega_{RF} & 0 \\ 0 & -\omega_{RF} & -\omega_Q - \omega_{Q22} & -\frac{\sqrt{3}}{2} \omega_{RF} \\ 0 & 0 & -\frac{\sqrt{3}}{2} \omega_{RF} & \omega_Q - \omega_{Q21} \end{pmatrix}$ ;

{HT, Tp} = Eigensystem[N[Ha]]; T = Transpose[Tp];
n1 = DiagonalMatrix[Exp[-i * Δt * HT]];
ρ1 = T.n1.Tp; ρ2 = T.Conjugate[n1].Tp; ρ0 = ρ1.ρ0.ρ2;
s[m] = ρ0;
]; (* End of For m *)

); (* End of sub-function f[] *)

(* Main function crystalMAS[] *)
crystalMAS[order_, ω0Mhz_, QCCMHz_, ωRFkHz_,

VrotkHz_, tf_, tau_, η_, αd_, βd_, γd_, quanta_] := {

ω0 = ω0Mhz * 2 π * 103; QCCbis = QCCMHz * 2 π * 103; ωbRFbis = ωRFkHz * 2 π;
ωrot = VrotkHz * 2 π; Δt = tau * 10-3; ns = tf / tau;
α = αd * π / 180; β = βd * π / 180; γ = γd * π / 180;
(* α ∈ [0, 2π] *) (* β ∈ [0, π] *) (* γ ∈ [0, 2π] *)
W00 = ( $\sqrt{5} / 10$ ) (3 + η2);

(* Table h stores the line intensity for each pulse duration *)
For [i = 0, i ≤ ns, i++, h[i] = 0;]; (* Clear Table h *)

(* Thermodynamic equilibrium of the density matrix *)
ρ0 = DiagonalMatrix[ $\left\{ \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2} \right\}$ ];

cα = Cos[α]; sα = Sin[α];
c2α = Cos[2 α]; s2α = Sin[2 α];
c4α = Cos[4 α];
cβ = Cos[β]; sβ = Sin[β];
c2β = Cos[2 β]; s2β = Sin[2 β];

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c4β = Cos[4 β];    s4β = Sin[4 β];
cγ = Cos[γ];      sγ = Sin[γ];
c2γ = Cos[2 γ];   s2γ = Sin[2 γ];
c3γ = Cos[3 γ];   s3γ = Sin[3 γ];
c4γ = Cos[4 γ];   s4γ = Sin[4 γ];

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(* Coefficients ai and bi involved in V(2,0) *)
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a1 = -η s2α sβ / √3;      b1 = -(-3 + η c2α) s2β / (2 √3);
a2 = -η cβ s2α / √6;     b2 = -(η c2α (3 + c2β) + 6 sβ2) / (4 √6);
d2s = a2 c2γ + b2 s2γ;   d1s = a1 cγ + b1 sγ;
d2c = a2 s2γ - b2 c2γ;   d1c = a1 sγ - b1 cγ;

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If [order == 2, {
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  (* Coefficients a2i and b2i involved in W(2,0) *)
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a22 = -√(2/7) η cβ s2α;   b22 = -(η c2α (3 + c2β) + sβ2 (-3 + η2)) / (2 √14);
a21 = -(2/√7) η sβ s2α;   b21 = (-3 - 2 c2α η + η2) s2β / (2 √7);
d22s = a22 c2γ + b22 s2γ; d21s = a21 cγ + b21 sγ;
d22c = a22 s2γ - b22 c2γ; d21c = a21 sγ - b21 cγ;

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  (* Coefficients a4i and b4i involved in W(4,0) *)
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a40 =
  -√(7/10) / 2304 ((18 + η2) (9 + 20 c2β + 35 c4β) + 240 η c2α (5 + 7 c2β) sβ2 + 280 η2 c4α sβ4);
a41 = (√(5/7) / 72) η s2α sβ (15 + 21 c2β + 14 η c2α sβ2);
b41 = (√(5/7) / 288) ((-18 - η2 - 12 η c2α + 7 η2 c4α) s2β - 7 (-3 + η c2α)2 s4β);
a42 = -(√(5/14) / 18) η cβ s2α (-9 + 21 c2β + 14 η c2α sβ2);
b42 =
  -1 / 72 √(5/14) (3 η c2α (5 + 4 c2β + 7 c4β) + (7 η2 c4α (3 + c2β) + (18 + η2) (5 + 7 c2β)) sβ2);
a43 = -(√(35) / 72) η (-3 - 9 c2β + η c2α (5 + 3 c2β)) s2α sβ;
b43 = -(√(35) / 288) (-18 - η2 - 12 η c2α + 7 η2 c4α + 2 (-3 + η c2α)2 c2β) s2β;
a44 = -(√(35/2) / 72) η cβ s2α (η c2α (3 + c2β) + 6 sβ2);
b44 = -√(35/2) / 2304 (η2 c4α (35 + 28 c2β + c4β) + 48 η c2α (3 + c2β) sβ2 + 8 (18 + η2) sβ4);
d41s = a41 cγ + b41 sγ;   d41c = a41 sγ - b41 cγ;   d42s = a42 c2γ + b42 s2γ;
d42c = a42 s2γ - b42 c2γ; d43s = a43 c3γ + b43 s3γ;   d43c = a43 s3γ - b43 c3γ;
d44s = a44 c4γ + b44 s4γ; d44c = a44 s4γ - b44 c4γ;
}]; (* End of If order == 2 *)

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f[order, QCCbis, ωBRFBis, Δt, ns]; (* Call the sub-function f[] *)
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For [i = 1, i ≤ ns, i++,
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  (* Normalized central-transition line intensity *)
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  If[quanta == 1, {s[i] = Im[s[i][[3, 2]]], h[i] = 2 s[i] / 5};];
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  (* -3-quantum line intensity *)
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  If[quanta == 3, {s[i] = Im[s[i][[4, 1]]], h[i] = s[i]};];
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];
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(*----- Provide Table crystalMAS containing -----*)
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(*-----pulse duration t and line intensity -----*)
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Print["*****"]; 
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For [a = 0, a ≤ ns, a++, time[a] = a * tau];
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crystalMAS = Chop[Table[{tt, time[tt], NumberForm[h[tt], 10]}, {tt, 0, ns}]];
Print[TableForm[crystalMAS,
  TableHeadings -> {None, {"Rang", "t( $\mu$ s)", "intensity"}}]];
(*----- Graph display -----*)
Print["*****"];
ListPlot[Table[{tt*tau, h[tt]}, {tt, 0, ns}],
  PlotJoined -> True,
  PlotLabel -> "Int=f(t)",
  AxesLabel -> {"t( $\mu$ s)", "Int.(U.A.)"},
  PlotStyle -> {Hue[0.1]},
  TextStyle -> {FontFamily -> "Times", FontSize -> 12}];
); (* End of main function crystalMAS[] *)

(* Call the main function with the corresponding numerical parameters *)
crystalMAS[ 2 , 105.8731007 , 8 ,
100 , 15 , 20 , 1 , 1 , 30 , 30 , 30 , 1 ];
(* crystalMAS[order_,  $\omega$ 0Mhz_, QCCMHz_,  $\omega$ RFkHz_,
VrotkHz_, tf_, tau_,  $\eta$ _,  $\alpha$ d_,  $\beta$ d_,  $\gamma$ d_, quanta_ ] *)

(*-----*)
(* Table crystalMAS.m in Microsoft EXCEL format *)
(*-----*)
Clear[writeExcel];
writeExcel[filename_String, data_List] :=
Module[ { file = OpenWrite[filename] },
  Scan[ (
    WriteString[file, First[#]];
    Scan[
      WriteString[file, "\t", #] &,
      Rest[#]
    ]; (* End of Scan *)
    WriteString[file, "\n"]
  ) &,
  data
]; (* End of Scan *)
Close[file]
]; (* End of Module *)

writeExcel["crystalMAS.m", crystalMAS];

Remove[order,  $\omega$ 0Mhz, QCCMHz,  $\omega$ RFkHz, VrotkHz, tf, tau,  $\eta$ ,  $\alpha$ d,  $\beta$ d,  $\gamma$ d, QCCbis,  $\omega$ bRFbis,
ns,  $\alpha$ ,  $\beta$ ,  $\gamma$ , i, h, f, a, crystalMAS, n, s, m,  $\omega$ Q, Ha, T, Tp, HT,  $\omega$ Q21,  $\omega$ Q22, quanta]

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